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Stochastic convective heat transfer equations in finite differences method

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Abstract

A numerical procedure based on the stochastic finite differences method is developed for the analysis of general problems in free/forced convection heat transfer. The discretization of the field equations through use of the finite differences approximation method is described. One-dimensional axisymmetrical problem is solved as an example.

1. Introduction

The analysis of heat and fluid flow whether subjected to random or deterministic external boundary conditions has been developed mainly under the assumption that the medium parameters are deterministic quantities.

For a significant number of circumstances, this assumption is not valid, and the probabilistic aspects of the medium need to be taken into account. In order to solve numerically convective heat transfer equations one uses the finite element and finite differences methods.

The application of numerical methods with probabilistic context leads to two classes of methods: first and second-order moment methods and reliability methods [1,4]. This paper addresses only the first category.

The finite difference method is used in the present investigations. Combining the finite differences method with stochastic analysis leads to stochastic finite differences method.

The purpose of the present paper is to describe and illustrate the use of stochastic finite differences method for the analysis of convective heat transfer problems. The stochastic finite differences method are rarely found in the literature. The most studied stochastic finite element method [2,3,6,7,9–12].

2. Basic equations

The development presented here is concerned combined convective and conductive transfer of thermal energy in regions containing a moving fluid [5,13]. The geometry of the fluid is limited to two-dimensions. The shape at the boundary is arbitrary. The fluid is assumed to be Newtonian are incompressible within the Boussinesq approximation. The flow is assumed to be time-independent and laminar. Material properties, such as viscosity and thermal conductivity can be assumed to vary with temperature. In this study, independence at these qualities on temperature is considered. The effects of viscous dissipation and radiative transport have been neglected in the present develop-

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ment. To simplify the following derivation, only the case at plane two-dimensional flow will be treated in detail.

Consider the previously indicated assumptions and restrictions, the appropriate mathematical description of the fluid motion is given by Navier–Stokes equations:

Energy: $$\rho c_p u_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j}\left(k_{ij}\frac{\partial T}{\partial x_j}\right) + q_v \qquad (1)$$

Continuity: $$\frac{\partial u_i}{\partial x_i} = 0 \qquad (2)$$

Momentum: $$\rho u_j \frac{\partial u_i}{\partial x_j} = \frac{\partial \tau_{ij}}{\partial x_j} - \rho g \beta (T - T_0) \qquad (3)$$

Constitutive: $$\tau_{ij} = p\delta_{ij} + \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \qquad (4)$$

Supplementary equation: $$\rho = \rho_0\left[1 - \beta(T - T_0)\right] \qquad (5)$$

where:

u_j is the velocity of fluid flow in direction x_i
x_j is the coordinate
ρ_0 is the density at reference temperature
c_p is the specific heat at constant pressure
k_{ij} is the thermal conductivity tensor
q_v is the rate of heat generated per unit volume
g is the acceleration due to gravity
T is the temperature
p is the pressure
μ is the fluid (laminar) viscosity
β is the coefficient of thermal expansion
δ_{ij} is the Kronecker delta

subscript 0 is the reference conditions

To complete the formulation of the boundary-value problem, suitable boundary conditions for the dependent variables are required. For the hydrodynamic part of the problem either velocity components or the total surface stress (or traction) must be specified on the boundary of the fluid region. The thermal part of the problem requires temperature or heat flux to be specified on all parts at the boundary. Symbolically, these conditions are expressed by:

$$u_i = f_i(s) \quad \text{on } S_u \qquad (6)$$

$$t_i = T_{ij}(s) n_j(s) \quad \text{on } S_t \qquad (7)$$

where $S = S_v \cup S_t$. In Eqs. (6) and (17) s denotes a generic point on the boundary, and n_j are the components of the outward unit normal to the boundary.

3. Finite differences approximation

The finite differences approximation of Eqs. (1)–(5) will be based on introducing the domain $D \cup S$ of a rectangular nonuniform grid with steps equals to $h_i^{x_1}$ and $h_j^{x_2}$ along x_1, x_2 axis, respectively.

Eqs. (1)–(5) are reduced to the following matrix form:

$$\mathbf{K}\boldsymbol{T} + \mathbf{L}(\boldsymbol{u})\boldsymbol{T} = \boldsymbol{F} \qquad (8)$$

$$\mathbf{A}\boldsymbol{u} = 0 \qquad (9)$$

$$\mathbf{R}(\boldsymbol{u})\boldsymbol{u} + \mathbf{D}\boldsymbol{u} = \boldsymbol{H}(\boldsymbol{T}) + \mathbf{B}\boldsymbol{P} \qquad (10)$$

where $\boldsymbol{T}$ is the vector of unknown temperatures at the grid nodes, $\boldsymbol{u}$ is the vector of unknown pair of velocities (u_1, u_2) at the grid nodes, $\boldsymbol{P}$ is the vector of unknown pressures at grid nodes.

The matrix $\mathbf{K}$ is called the conductivity matrix and is dependent on thermal conductivity coefficient k

$$\mathbf{K} = \mathbf{K}(k) \qquad (11)$$

The matrix $\mathbf{L}$ is dependent on density ρ and heat capacity c_p

$$\mathbf{L} = \mathbf{L}(\rho, c_p) \qquad (12)$$

The matrix $\mathbf{R}$ is dependent on density ρ

$$\mathbf{R} = \mathbf{R}(\rho) \qquad (13)$$

The matrix $\mathbf{D}$ is dependent on viscosity μ

$$\mathbf{D} = \mathbf{D}(\mu) \qquad (14)$$

The vector $\boldsymbol{H}$ is dependent on density ρ, gravity coefficient g and volumetric expansion β

$$\boldsymbol{H} = \boldsymbol{H}(\rho, g, \beta) \qquad (15)$$

4. Stochastic finite difference approximation

Stochastic finite difference matrix equations of the problem Eqs. (8)–(10) will be derived by assuming that the material parameters are functions of random variable vector:

$$\boldsymbol{b} = (b_1, b_2, \ldots, b_R) \qquad (16)$$

Eqs. (8)–(10) with the above assumptions take the form:

$$\mathbf{K}(\boldsymbol{b})\boldsymbol{T}(\boldsymbol{b}) + \mathbf{L}\big(\boldsymbol{u}(\boldsymbol{b}), \boldsymbol{b}\big)\boldsymbol{T}(\boldsymbol{b}) = \boldsymbol{F}(\boldsymbol{b}) \qquad (17)$$

$$\mathbf{A}\boldsymbol{u}(\boldsymbol{b}) = 0 \tag{18}$$

$$\mathbf{R}\big(\boldsymbol{u}(\boldsymbol{b}), \boldsymbol{b}\big)\boldsymbol{u}(\boldsymbol{b}) + \mathbf{D}(\boldsymbol{b})\boldsymbol{u}(\boldsymbol{b}) = \boldsymbol{H}\big(\boldsymbol{T}(\boldsymbol{b}), \boldsymbol{b}\big) + \mathbf{B}(\boldsymbol{b})\boldsymbol{P}(\boldsymbol{b}) \tag{19}$$

The random function $b(\boldsymbol{x})$ is approximated using shape functions $N_i(x)$ by

$$b(\boldsymbol{x}) = \sum_{i=1}^{q} N_i(x) b_i = \boldsymbol{N}\boldsymbol{b} \tag{20}$$

where b_i are the nodal values of $b(\boldsymbol{x})$, that is the values of b at x_i, $i = 1, \ldots, q$.

The mean value of $\boldsymbol{b}$ denoted by $E(\boldsymbol{b})$ is expressed as

$$E(\boldsymbol{b}) = \sum_{i=1}^{b} N_i E(b_i) \tag{21}$$

and the variance by

$$V(\boldsymbol{b}) = \alpha^2 E(\boldsymbol{b})^2 \tag{22}$$

where α is the coefficient of variation.

All the random functions are expended about the mean value $E(b)$ via a Taylor series and only up to second-order terms are retained for any small parameter γ we have:

$$\mathbf{T}(\boldsymbol{b}) = E(\mathbf{T}) + \gamma \sum_{i=1}^{q} E(\mathbf{T})_{,b_i} \Delta b_i + \frac{1}{2}\gamma^2 \sum_{i,\, j=1}^{q} E(\mathbf{T})_{,b_i b_j} \Delta b_i\, \Delta b_j \tag{23}$$

$$\mathbf{K}(\boldsymbol{b}) = E(\mathbf{K}) + \gamma \sum_{i=1}^{q} E(\mathbf{K})_{,b_i} \Delta b_i + \frac{1}{2}\gamma^2 \sum_{i,\, j=1}^{q} E(\mathbf{K})_{,b_i b_j} \Delta b_i\, \Delta b_j \tag{24}$$

$$\mathbf{F}(\boldsymbol{b}) = E(\mathbf{F}) + \gamma \sum_{i=1}^{q} E(\mathbf{F})_{,b_i} \Delta b_i + \frac{1}{2}\gamma^2 \sum_{i,\, j=1}^{q} E(\mathbf{F})_{,b_i b_j} \Delta b_i\, \Delta b_j \tag{25}$$

$$\mathbf{u}(\boldsymbol{b}) = E(\mathbf{u}) + \gamma \sum_{i=1}^{q} E(\mathbf{u})_{,b_i} \Delta b_i + \frac{1}{2}\gamma^2 \sum_{i,\, j=1}^{q} E(\mathbf{u})_{,b_i b_j} \Delta b_i\, \Delta b_j \tag{26}$$

$$\mathbf{D}(\boldsymbol{b}) = E(\mathbf{D}) + \gamma \sum_{i=1}^{q} E(\mathbf{D})_{,b_i} \Delta b_i + \frac{1}{2}\gamma^2 \sum_{i,\, j=1}^{q} E(\mathbf{D})_{,b_i b_j} \Delta b_i\, \Delta b_j \tag{27}$$

For any function $g(G(b), b)$ we have:

$$\begin{aligned} g\big(G(b), b\big) = {} & E(g) + \gamma \sum_{i=1}^{q} E\big(g_{,b_i} + g_{,G} G_{,b_i}\big)\Delta b_i \\ & + \frac{1}{2}\gamma^2 \sum_{i,\, j=1}^{q} E\big(g_{,b_i b_j} + g_{,G b_j} G_{,b_i} \\ & + g_{,G} G_{,b_i b_j} + g_{,b_i G} G_{,b_j} \\ & + g_{,GG} G_{,b_j} G_{,b_j}\big)\Delta b_i\, \Delta b_j \end{aligned} \tag{28}$$

With the assumptions that $g = g^1(G(b)) + g^2(b)$ and $g^1(G(b))$ is linear function of G we get from Eq. (28):

$$\begin{aligned} g\big[G(b), b\big] = {} & E(g) + \gamma \sum_{i=1}^{q} E\big(g_{,b_i} + g_{,G} G_{,b_i}\big)\Delta b_i \\ & + \frac{1}{2}\gamma^2 \sum_{i,\, j=1}^{q} E\big(g_{,b_i b_j} + g_{,G} G_{,b_i b_j}\big)\Delta b_i\, \Delta b_j \end{aligned} \tag{29}$$

Using Eq. (29) we get the following relations:

$$\begin{aligned} \mathbf{H}\big(\boldsymbol{T}(\boldsymbol{b}), \boldsymbol{b}\big) = {} & E(\mathbf{H}) + \gamma \sum_{i=1}^{q} E\big(\mathbf{H}_{,b_i} + \mathbf{H}_{,T} \boldsymbol{T}_{,b_i}\big)\Delta b_i \\ & + \frac{1}{2}\gamma^2 \sum_{i,\, j=1}^{q} E\big(\mathbf{H}_{,b_i b_j} \\ & + \mathbf{H}_{,T} \boldsymbol{T}_{,b_i b_j}\big)\Delta b_i\, \Delta b_j \end{aligned} \tag{30}$$

$$\begin{aligned} \mathbf{R}\big(\boldsymbol{u}(\boldsymbol{b}), \boldsymbol{b}\big) = {} & E(\mathbf{R}) + \gamma \sum_{i=1}^{q} E\big(\mathbf{R}_{,b_i} + \mathbf{R}_{,u} \boldsymbol{u}_{,b_i}\big)\Delta b_i \\ & + \frac{1}{2}\gamma^2 \sum_{i,\, j=1}^{q} E\big(\mathbf{R}_{,b_i b_j} + \mathbf{R}_{,u} \boldsymbol{u}_{,b_i b_j}\big)\Delta b_i \Delta b_j \end{aligned} \tag{31}$$

$$\begin{aligned} \mathbf{L}\big(\boldsymbol{u}(\boldsymbol{b}), \boldsymbol{b}\big) = {} & E(\mathbf{L}) + \gamma \sum_{i=1}^{q} E\big(\mathbf{L}_{,b_i} + \mathbf{L}_{,u} \boldsymbol{u}_{,b_i}\big)\Delta b_i \\ & + \frac{1}{2}\gamma^2 \sum_{i,\, j=1}^{q} E\big(\mathbf{L}_{,b_i b_j} + \mathbf{L}_{,u} \boldsymbol{u}_{,b_i b_j}\big)\Delta b_i\, \Delta b_j \end{aligned} \tag{32}$$

where Δb_i represents the first-order variation of b_i about $E(b_i)$ and for any function g

$$E(g(x)) = g\big(x, E(b)\big) \tag{33}$$

$$E(g_{,b_1}) = \frac{\partial g}{\partial b_1} \tag{34}$$

$$E(g_{,b_1 b_2}) = \frac{\partial^2 g}{\partial b_1 \partial b_2} \tag{35}$$

Substitution of Eqs. (30)–(32) and (23)–(27) into Eqs.

(17)–(19) and collecting terms of order 1, γ and γ^2, we arrive:

Zeroth-order

$$E(\mathbf{K})E(\mathbf{T}) + E(\mathbf{L})E(\mathbf{T}) = E(\mathbf{F}) \tag{36}$$

$$E(\mathbf{A})E(\mathbf{u}) = 0 \tag{37}$$

$$E(\mathbf{R})E(\mathbf{u}) + E(\mathbf{D})E(\mathbf{u}) = E(\mathbf{H}) + E(\mathbf{B})E(\mathbf{P}) \tag{38}$$

First-order

$$E(\mathbf{K})E(\mathbf{T})_{,b_i} + E(\mathbf{L})E(\mathbf{T})_{,b_i} = E(\mathbf{F})_{,b_i} - \left[E(\mathbf{K})_{,b_i}E(\mathbf{T}) + E(\mathbf{L})_{,b_i}E(\mathbf{T})\right] \tag{39}$$

$$E(\mathbf{A})E(\mathbf{u})_{,b_i} = -E(\mathbf{A})_{,b_i}E(\mathbf{u}) \tag{40}$$

$$E(\mathbf{R})E(\mathbf{u})_{,b_i} + E(\mathbf{D})E(\mathbf{u})_{,b_i} = E\left(\mathbf{H}_{,b_i} + \mathbf{H}_{,T}\mathbf{T}_{,b_i}\right) + E(\mathbf{B})_{,b_i}E(\mathbf{P})_{,b_i} - \left[E(\mathbf{R})_{,b_i}E(\mathbf{u}) + E(\mathbf{D})_{,b_i}E(\mathbf{u})\right] \tag{41}$$

Second-order

$$E(\mathbf{K})E(\hat{\mathbf{T}}_2) + E(\mathbf{L})E(\hat{\mathbf{T}}_2) = \frac{1}{2}\sum_{i,\,j=1}^{q}\left[E(\mathbf{F})_{,b_ib_j}\right]\mathrm{Cov}(b_i, b_j) - \sum_{i,\,j=1}^{q}\left[\frac{1}{2}E(\mathbf{K})_{,b_ib_j}E(\mathbf{T}) + \frac{1}{2}E(\mathbf{L})_{,b_ib_j}E(\mathbf{T}) + E(\mathbf{K})_{,b_i}E(\mathbf{T})_{,b_j} + E(\mathbf{L})_{,b_i}E(\mathbf{T})_{,b_j}\right]\mathrm{Cov}(b_i, b_j) \tag{42}$$

where

$$\hat{\mathbf{T}}_2 = \frac{1}{2}\sum_{i,\,j=1}^{q}E(\mathbf{T})_{,b_ib_j}\mathrm{Cov}(b_i, b_j) \tag{43}$$

$$E(\mathbf{A})E(\hat{\mathbf{u}}_2) = -\sum_{i,\,j=1}^{q}\left[\frac{1}{2}E(\mathbf{A})_{,b_ib_j}E(\mathbf{u}) - E(\mathbf{A})_{,b_i}E(\mathbf{u})_{,b_j}\right]\mathrm{Cov}(b_i, b_j) \tag{44}$$

$$E(\mathbf{R})E(\hat{\mathbf{u}}_2) + E(\mathbf{D})E(\hat{\mathbf{u}}_2) = \frac{1}{2}\sum_{i,\,j=1}^{q}\left[E\left(\mathbf{H}_{,b_ib_j} + \mathbf{H}_{,T}\boldsymbol{T}_{,b_ib_j}\right) + \frac{1}{2}E(\mathbf{B})_{,b_ib_j}E(\mathbf{P})_{,b_ib_j}\right]\mathrm{Cov}(b_i, b_j) - \sum_{i,\,j=1}^{q}\left[E\left(\mathbf{R}_{,b_i} + R_{,u}\boldsymbol{u}_{,b_i}\right)E(\mathbf{u})_{,b_j} + E(\mathbf{D})_{,b_i}E(\mathbf{u})_{,b_j}\right]\mathrm{Cov}(b_i, b_j) - \frac{1}{2}\sum_{i,\,j=1}^{q}\left[E\left(\mathbf{R}_{,b_ib_j} + \mathbf{R}_{,T}\boldsymbol{T}_{,b_ib_j}\right)E(\mathbf{u}) + E(\mathbf{D})_{,b_ib_j}E(\mathbf{u})\right]\mathrm{Cov}(b_i, b_j) \tag{45}$$

where

$$\hat{\mathbf{u}}_2 = \frac{1}{2}\sum_{i,\,j=1}^{q}E(\hat{\mathbf{u}}_2)_{,b_ib_j}\mathrm{Cov}(b_i, b_j)$$

$$\mathrm{Cov}(b_i, b_j) = \left[V(b(x_i))V(b(x_j))\right]^{\frac{1}{2}}R(b(x_i), b(x_j)) \tag{46}$$

and $R(b(x_i), b(x_j))$ is the autocorrelation.

5. Expectation values of temperature and velocity

The definitions for the expectation and autocovariance of the temperature are given by

$$E(\boldsymbol{T}) = \int_{-\infty}^{+\infty}\boldsymbol{T}(\boldsymbol{b}, t)f(\boldsymbol{b})\,\mathrm{d}\boldsymbol{b} \tag{47}$$

and

$$\mathrm{Cov}(T^i, T^j) = \int_{-\infty}^{+\infty}(T^i - E(T^i))(T^j - E(T^j))f(\boldsymbol{b})\,\mathrm{d}\boldsymbol{b} \tag{48}$$

where $f(\boldsymbol{b})$ is the joint probability density function.

The second-order estimate of the mean value of $\boldsymbol{T}$ is obtained from Eq. (1) to give

$$E(\boldsymbol{T}) = \boldsymbol{T}(E(\boldsymbol{b})) + \frac{1}{2}\left\{\sum_{i,\,j=1}^{q}E(\boldsymbol{T}_{b_i,\,b_j})\mathrm{Cov}(b_i, b_j)\right\} \tag{49}$$

The similar definition is used for the expectation of the velocity

$$E(\boldsymbol{u}) = \int_{-\infty}^{+\infty} \boldsymbol{u}(\boldsymbol{b}, t) f(\boldsymbol{b})\, \mathrm{d}\boldsymbol{b} \tag{50}$$

and for autocovariance

$$\mathrm{Cov}(\boldsymbol{u}^i, \boldsymbol{u}^j) = \int_{-\infty}^{+\infty} (\boldsymbol{u}^i - E(\boldsymbol{u}^i)) \times (\boldsymbol{u}^j - E(\boldsymbol{u}^j)) f(\boldsymbol{b})\, \mathrm{d}\boldsymbol{b} \tag{51}$$

6. Example

Consider problem at combined heat transfer and fluid flow in tube of inner radius $r_1 = 10$ cm and an outer radius $r_2 = 18$ cm (Fig. 1).

The problem is described by the following equations:

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}u}{\mathrm{d}r}\right) + g\beta T = 0 \tag{52}$$

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}T}{\mathrm{d}r}\right) = 0 \tag{53}$$

$$\mathrm{div}\, u = 0 \tag{54}$$

with the following boundary conditions:

$$u_{r=r_1} = 0$$

$$u_{r=r_2} = 0$$

$$T_{r=r_1} = T_1$$

$$T_{r=r_2} = T_2$$

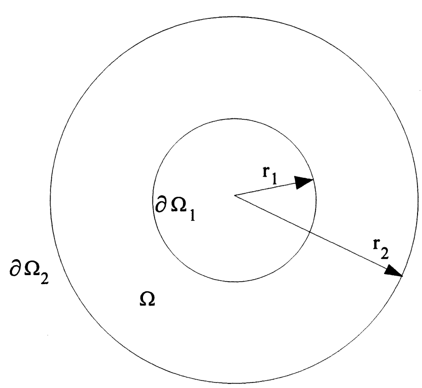

Fig. 1. Scheme of axisymmetrical thermal problem.

Table 1
Expectations of temperatures

Radius (cm)	10	12	14	16	18
Temperature (°C)	5.00	4.07	3.28	2.60	2.00
Standard deviation	1.22	1.02	0.79	0.63	0.47

The following random data are adopted:

$$E(T_1) = 5$$

$$E(T_2) = 2$$

$$E(g) = 1$$

$$E(\beta) = 1$$

cross-correlation functions

$$\mu(g^\rho, g^\sigma) = \exp[-\mathrm{abs}(r_i - r_j)/\xi_g]$$

$$\mu(\beta^\rho, \beta^\sigma) = \exp[-\mathrm{abs}(r_i - r_j)/\xi_\beta]$$

with correlation lengths $\xi_g = \xi_\beta = 1$ and coefficient of variation $\alpha_g^\sigma = \alpha_\beta^\sigma = 0.15$.

Numerical results as to expectations of temperatures and velocities are shown in Tables 1 and 2.

The solution is obtained by an implicit finite difference technique with a constant grid and variable time step sizes. A mesh size 2×10^{-2} was found to be adequate for the example. Since equations given above are of parabolic type, the solutions of such equations are well known and can be found in many text books on numerical analysis.

7. Concluding remarks

The proposed numerical method allows one to calculate temperature and velocity fields of expected values. Stochastic mathematical model contains material and boundary coefficients, which are random functions of coordinates. The discrete analog of the stochastic

Table 2
Expectations of velocities

Radius (cm)	10	12	14	16	18
Velocity $\times 10^{-3}$ (cm/s)	0	6.24	8.22	2.13	0
Standard deviation	0	1.53	2.01	0.53	0

mathematical model is obtained by the method of finite differences.

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